THE INVESTIGATION OF NONSTATIONARY HEAT TRANSFER BY ELECTRIC MODELS

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A device is discussed which can extend the potentialities of existing models and permit the solution of problems in nonstationary heat conduction with time-dependent boundary conditions of the third kind.

At the present time electrical models, RC-networks, which belong to the class of continuous analog mathematical machines (AMM), are widely used to solve problems in nonstationary heat conduction. In spite of the known advantages of RC-networks over other models, the solution of nonstationary heat conduction problems with time-dependent boundary conditions of the third kind on them is difficult. This is because there are no devices for the continuous simulation of one component of the boundary conditions – the time-dependent heat-transfer coefficients $\alpha = f(\tau)$, the electrical analog of which is the conductance $(1/R_\alpha)$.

Such problems can be solved on existing AMM, for example, on an USM-1 [1] if we use a block which reproduces the step-approximation function $\alpha_s = \varphi(\tau)$ [2] to simulate the function $\alpha = f(\tau)$. It was proved in [3] that it is permissible to replace continuous functions of the form $\alpha = f(\tau)$ by step functions when α is constant in a bounded number of intervals. It should be noted that the possible wide use of such a block is restricted by the complexity of its circuit and also by the labor in calculating its parameters and the duration of the program for each given function $\alpha = f(\tau)$.

Below we discuss a device which can be used with RC-networks to solve boundary-value problems of the third kind when the boundary conditions vary continuously with time, which until recently has been impossible. The operation of the proposed device is based on a new method of specifying the boundary conditions of the third kind which makes it possible to eliminate from the model the boundary resistor R_α and to simulate the heat-transfer coefficients by a voltage U_α which is linear in time with constant α. We know that the construction of voltages varying arbitrarily with the time is not difficult and to this end, in existing AMM various function generators (FG) are used.

At the present time to implement in a model the boundary condition of the third kind

$$\alpha (T_{me} - T_s) = -\lambda \frac{\partial T}{\partial n} \tag{1}$$

TABLE 1. The Temperature Field of an Unbounded Flat Plate for $\overline{\theta}_m = 1$; $\overline{\theta}_0 = 0.4$; Bi = 1 + 0.5 Fo – 0.5 exp (–Fo)

Fo	$\overline{\theta}(0;\ Fo)$			$\overline{\theta}(0.5;\ Fo)$			$\overline{\theta}(1;\ Fo)$		
	Vanichev's method	Vidin's method	DSVBC	Vanichev's method	Vidin's method	DSVBC	Vanichev's method	Vidin's method	DSVBC
0,25			0,522			0,455			0,432
0,50	0,658	0,664	0,648	0,536	0,539	0,530	0,497	0,495	0,490
0,75	0,734	0,734	0,721	0,617	0,617	0,610	0,577	0,575	0,576
1,00	0,797	0,794	0,789	0,691	0,688	0,685	0,654	0,650	0,656
1,25			0,849			0,732			0,719
1,50	0,887	0,882	0,885	0,811	0,806	0,805	0,784	0,773	0,780
1,75			0,914			0,857			0,839
2,00	0,940	0,936	0,940	0,892	0,887	0,888	0,870	0,868	0,868
2,25			0,959			0,922			0,912
2,50	0,970	0,966	0,966	0,941	0,937	0,935	0,931	0,926	0,932

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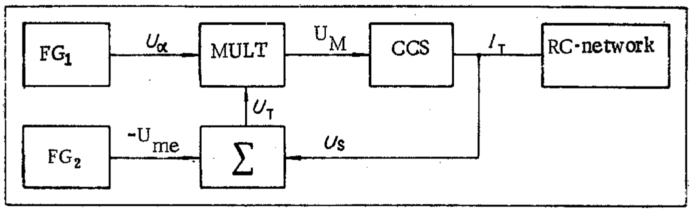

Fig. 1. Block diagram of a device for specify variable boundary conditions of the third kind (DSVBC).

between the potential U_{me}, corresponding to T_{me}, and the potential of a boundary point U_s, corresponding to T_s, a resistor R_α is included. A current I_T, defined by the equation

$$I_T = K_T\alpha(T_{me} - T_s) = \frac{1}{R}(U_{me} - U_s), \tag{2}$$

flows through R_α, where

$$U_s = K_t T_s;\ U_s = K_t T_s;\ R_\alpha = K_R\frac{1}{\alpha}.$$

The resistor R_α in (2) can be eliminated if we use the voltage U_α to simulate the coefficients α. Then the expression for the current I_T takes the form

$$I_T = \frac{1}{K_\alpha K_R}U_\alpha(U_{me} - U_s), \tag{3}$$

where

$$U_\alpha = K_\alpha\alpha = K_\alpha K_R\frac{1}{R_\alpha}.$$

Thus we have shown that a boundary condition of the third kind can be implemented by a current at a boundary point of the model proportional to the product of the voltage U_α and the difference between the voltages U_{me} and U_s. To form the current I_T the electrical model must have electronic blocks to implement the mathematical operations of multiplication and addition and also the operation of transforming voltage into current. It should, however, be noted that even if these blocks are present, I_T cannot be formed by the known methods in use at the present time in analog models, since one component (U_s) is unknown and has to be determined during the solution.

Figure 1 shows the block diagram of the proposed device to specify variable boundary conditions of the third kind (DSVBC), the essential distinction of which is that it is constructed on electrical simulation principles. The DSVBC contains two function generators FG_1 and FG_2, an adder Σ, a multiplier MULT, and a controllable current stabilizer CCS.

The purpose of the function generators is to form the voltages U_{me} and U_α which vary with time according to known rules.

The multiplication block is designed to multiply two continuously varying voltages, one of which (U_α) is input from FG_1 and the other ($U_T = U_{me} - U_s$) is input from the adder. The output of MULT is

$$U_m = K_m U_\alpha U_T = K_m U_\alpha(U_{me} - U_s), \tag{4}$$

which is the input of CCS.

The CCS is designed to transform U_M into the current proportional to it

$$I_T = K_i U_M = K_i K_m U_\alpha(U_{me} - U_s) \tag{5}$$

and to provide this current at a boundary point of the model.

The current I_T formed by the device is proportional both to the known variables α and T_{me}, and to the unknown variable T_s. To obtain the unknown voltage U_s in the DSVBC we use negative feedback, i.e., one of the inputs of the adder is connected to the boundary point of the model. It can be shown that, due to the feedback loop, the DSVBC is stable and the potential of the boundary point is U_s provided that

$$K_i K_m K_\alpha K_R = 1. \tag{6}$$

The calculation of the DSVBC parameters can be reduced to the determination of the transformation coefficients K_α, K_M, K_i, which, with K_R, satisfy Eq. (6).

The coefficient K_R is known since it is defined in the calculation of the RC-network parameters.

The coefficient K_α must be chosen from the condition that the range of variation of U_α must be maximal for given range of α ($K = \alpha_{max}/\alpha_{min}$). Then for α_{max} we must take 100% of the output voltage from FG_1. Thus

$$K_\alpha = \frac{100\%}{\alpha_{max}} \tag{7}$$

and U_α varies from $U_{\alpha_{min}} = 100\%/K$ to $U_{\alpha_{max}} = 100\%$. If we construct the function $\alpha = f(\tau)$ for the relative variable $\bar{\alpha} = \alpha/\alpha_0 = f(\tau)$, $\bar{\alpha}$ varies from 1 to 1/K. Then $K_\alpha = 100\%$.

The coefficient K_M for the multiplication block may be constant or variable. From the point of view of tuning the greatest number of channels it is expedient to choose K_M to be constant, and for USM-1, for example, equal to 0.05.

The coefficient K_i of the CCS is computed from the following equation when K_R, K_α, and K_M are known:

$$K_i = \frac{1}{K_R K_\alpha K_m}. \tag{8}$$

We see that K_i is variable; it changes with the conditions of the problem. Hence, the circuit of the CCS must allow K_i to vary smoothly within wide limits.

When the feedback link is disconnected from the output of the CCS, a current $I_{me} = K_T \alpha T_{me}$, known from the boundary conditions, must flow into the boundary point of the model. It is convenient to use this current to correct the coefficient K_i of the CCS and to check that the parameters of the DSVBC have been calculated correctly. The coefficient K_T is determined from the equation

$$K_\tau = \frac{K_t}{K_R}. \tag{9}$$

The essential advantage of the device discussed above is that the variables α and T_{me} are simulated by the voltages U_α and U_{me}, each of which is formed independently of the other by its FG and is an independent factor in the multiplication block. Hence in solving a problem with the model we can vary $\alpha = f(\tau)$ and $T_{me} = f(\tau)$ arbitrarily and so we can study the effect of α and T_{me} on nonstationary temperature fields and also solve inverse heat conduction problems and the problem of the optimal solution, which means that the AMM is more universal.

As a test the DSVBC was constructed from standard USM-1 blocks. The FG from that machine, GU-2 channels and also multiplication blocks using GU-1 channel amplifiers in a circuit forming one quarter of the multiplier with thyrite square-law function generators, were included. By using GU-1 channels in the circuit it was possible to avoid the need to add special multipication blocks to complement the USM-1. By using the device in conjunction with the USM-1, the continuous solution was obtained for the first time for a number of problems when the heat-transfer coefficients varied continuously with time.

As an example, Table 1 gives the results of solving problems discussed in (4) by Vanichev's numerical method, Vidin's analytic method, and by electrical simulation using the USM-1 and the DSVBC. The difference between the solutions does not exceed 1.5%, which confirms that it is possible to use the DSVBC in practice.

In conclusion we note that if α = const, the DSVBC is significantly simplified. It consists only of an FG forming the voltage U_{me} and a CCS with two added inputs, one of which is connected to the FG and the other is connected to the boundary point of the model, and so is connected to the output of the CCS.

NOTATION

T_{me} is the temperature of the medium, deg;
T_s is the surface temperature of body, deg;
α is the heat-transfer coefficient, $W/m^2 \cdot deg$;

λ is the coefficient of thermal conductivity, W/m·deg;

R_α is the boundary resistor, ohm;

I is the current, amp;

U is the potential, volts;

K_t is the transition coefficient from temperature to potential, V/deg;

K_R is the transition coefficient from thermal to electrical resistances, ohm·W/m^2·deg;

K_T is the transition coefficient from heat flow to electrical current, m^2/V;

K_α is the transition coefficient from α to U_α, m^2·deg/A;

K_M is the transformation coefficient for the multiplication block, 1/V;

K_i is the transformation coefficient for the CCS, A/V.

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